Journal of Engineering Physics and Thermophysics, Vol. 72, No. 2, 1999

CONDUCTIVE-CONVECTIVE HEAT TRANSFER IN DISPERSE SYSTEMS WITH SUSPENDED PARTICLES

Yu. S. Teplitskii

UDC 66.096.5

A film model of heat transfer between a disperse medium and a macrosurface under conditions of an ascending gas flow is suggested. Model equation (11) for the coefficient $\alpha_{c\text{-}c}$ as a function of the governing factors is obtained. It is used for generalization of experimental data in various systems: a fludized bed, a circulating fluidized bed, vertical pneumotransport — described by formulas (12), (13), (15), and (16).

Two-phase media in which particles are suspended by an ascending gas flow (a fluidized bed, a circulating fludized bed, vertical pneumotransport) are widely used in industry. A large number of works are devoted to investigation of heat transfer between such systems and macrosurfaces (especially for a fluidized bed), which is of importance in practice [1-5]. The correlations established for calculation of $\alpha_{c\text{-}c}$ have, as a rule, a rather limited domain of verification, which narrows the possibility of their practical use. In order to obtain simple and universal relations for calculation of $\alpha_{c\text{-}c}$ needed in practice, it is necessary to attain a sufficiently high level of generalization that is unthinkable without creating models that describe the most general significant features of the process of heat transfer between a two-phase medium and a macrosurface and that can predict the character of the dependence of $\alpha_{c\text{-}c}$ on the governing factors not only in a single system but also in an entire class of related systems. Among those are the fluidized bed, the circulating fluidized bed, and the vertical pneumotransport mentioned above. These systems are closely, related owing to their ability to convert from one to another with a smooth change in the gas velocity and to the fact that the excess gas filtration rate $u - u_t^*$ characterizes the intensity of transfer processes in all cases.

The analysis below is based on a film model of heat transfer that we developed for a fluidized bed in [6]. According to this model, which uses a fundamental property of a disperse medium (heat is transferred to it from the surface at first through a boundary gaseous film), the coefficient $\alpha_{c\text{-}c}$ is determined by the simple expression

$$\alpha_{c-c} = \lambda_{ef}/\delta_f , \tag{1}$$

which shows that $\alpha_{c\text{-}c}$ is completely determined by the thermal resistance of the boundary gaseous film. Effective values of the film thickness (δ_f) and the horizontal thermal conductivity (λ_{ef}) are expressed in terms of the physical characteristics of the two-phase system.

The model of heat transfer in disperse media with suspended particles is a generalization of [6] and is based on the following assumptions:

1) the thickness of the boundary gaseous film is a nonlinear function of the particle diameter and the particle concentration in the flow:

$$\delta_f/d = k_1 \,\mathrm{Ar}^{-k_2} (1 - \varepsilon)^{-k_3} ; \tag{2}$$

2) the temperature in the boundary gaseous film has a linear profile;

3) the effective thermal conductivity of the gaseous film consists of conductive and convective components:

$$\lambda_{ef} = \lambda_{cond} + \lambda_{conv} ; \tag{3}$$

Academic Scientific Complex "A. V. Luikov Heat and Mass Transfer Institute of the National Academy of Sciences of Belarus," Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 2, pp. 317-322, March-April, 1999. Original article submitted November 12, 1997.

1062-0125/99/7202-0297$22.00

4) the conductive component of λ_{ef} includes the effect of convection of particles penetrating into the near-surface zone and has the form

$$\lambda_{cond} = k_4 \lambda_f \left(\frac{c_s}{c_f}\right)^{k_5} \left(\frac{\rho_s}{\rho_f}\right)^{k_6} ; \tag{4}$$

5) the convective component of λ_{ef} takes into account the contribution of the filtering gas and the nonlinear dependence of λ_{conv} on d and $1 - \varepsilon$:

$$\lambda_{conv} = k_7 c_f \rho_f u_w d \left(\frac{d}{D}\right)^{k_8} \mathrm{Ar}^{-k_2^*} (1 - \varepsilon)^{-k_3^*} , \tag{5}$$

These rather general assumptions make it possible, as shown below, to obtain universal relations of the same type for calculation of $\alpha_{c\text{-}c}$ in the systems under consideration.

We write the equation of heat balance on the heat-transfer surface, which determines $\alpha_{c\text{-}c}$:

$$\alpha_{c-c} (T_\infty - T_w) = - \lambda_{ef} \left.\frac{\partial T}{\partial r}\right|_{r=D/2} . \tag{6}$$

Using assumption 2), we can write the following expression for the temperature gradient on the wall:

$$\left.\frac{\partial T}{\partial r}\right|_{r=D/2} = \frac{T_w - T_s}{\delta_f} . \tag{7}$$

Substituting this expression for $\partial T/\partial r$ into (6), we arrive at

$$\alpha_{c-c} = \lambda_{ef} \Theta_s / \delta_f , \tag{8}$$

which is a generalization of (1).

The dimensionless relative temperature Θ_s of the first series of particles characterizes the temperature profile in a horizontal cross section of the apparatus (standpipe). This quantity serves as an empirical parameter and considerably simplifies the heat-transfer model by allowing its consideration in a one-zone approximation. In a fluidized bed, owing to its high horizontal thermal conductivity the quantity Θ_s can be considered to be unity and formula (8) turns into (1). As an analysis [8] of experimental data [7] for a circulating fluidized bed and data obtained by Nosov ([5], Fig. 8-12] under pneumotransport conditions has revealed, the quantity Θ_s is a rather weak function of the particle concentration:

$$\Theta_s = k_9 (1 - \varepsilon)^{k_{10}} , \tag{9}$$

where $k_9 = 1.29$, $k_{10} = 0.13$ (a circulating fluidized bed) and $k_9 = 0.9$, $k_{10} = 0.18$ (pneumotransport).

Substituting into (8) its constituting quantities expressed in (2), (4), (5) and (9), we obtain

$$\alpha_{c-c} = \frac{k_4 k_9}{k_1} \frac{\lambda_f}{d} \mathrm{Ar}^{k_2} (1 - \varepsilon)^{k_3 + k_{10}} \left(\frac{c_s}{c_f}\right)^{k_5} \left(\frac{\rho_s}{\rho_f}\right)^{k_6} +$$

$$+ \frac{k_7 k_9}{k_1} c_f \rho_f u_w d \left(\frac{d}{D}\right)^{k_8} \mathrm{Ar}^{k_2 - k_2^*} (1 - \varepsilon)^{k_3 - k_3^* + k_{10}} . \tag{10}$$

In dimensionless form expression (10) is

$$\mathrm{Nu}_{c-c} = \frac{k_4 k_9}{k_1} \mathrm{Ar}^{k_2} (1 - \varepsilon)^{k_3 + k_{10}} \left(\frac{c_s}{c_f}\right)^{k_5} \left(\frac{\rho_s}{\rho_f}\right)^{k_6} +$$

$$+\frac{k_7 k_9}{k_1}\mathrm{Re_w}\,\mathrm{Pr}\left(\frac{d}{D}\right)^{k_8}\mathrm{Ar}^{k_2-k_2^*}(1-\varepsilon)^{k_3-k_3^*+k_{10}}. \qquad (11)$$

Equation (11) is the most general form of the dependence of $\mathrm{Nu_{c\text{-}c}}$ on the governing parameters. The excess filtration rate $u - u_t^*$ enters (11) indirectly in terms of the particle concentration $1 - \varepsilon$, which is a function of $u - u_t^*$.

Now we will consider model equation (11) as applied to particular disperse media.

1. Fluidized bed. As mentioned above, $\Theta_s = 1$. The Reynolds number $\mathrm{Re_w}$ is represented as $\mathrm{Re_w} = \mathrm{Re}/\varepsilon$. Equation (11) acquires the form

$$\mathrm{Nu_{c-c}} = \frac{k_4}{k_1}\mathrm{Ar}^{k_2}(1-\varepsilon)^{k_3}\left(\frac{c_s}{c_f}\right)^{k_5}\left(\frac{\rho_s}{\rho_f}\right)^{k_6} +$$

$$+\frac{k_7}{k_1}\frac{\mathrm{Re}}{\varepsilon}\mathrm{Pr}\left(\frac{d}{D}\right)^{k_8}\mathrm{Ar}^{k_2-k_2^*}(1-\varepsilon)^{k_3-k_3^*}. \qquad (11a)$$

In [6], based is an analysis of numerous experimental data, the following semiempirical equation was obtained:

$$\mathrm{Nu_{c-c}} = 0.74\,\mathrm{Ar}^{0.1}(1-\varepsilon)^{2/3}\left(\frac{c_s}{c_f}\right)^{0.24}\left(\frac{\rho_s}{\rho_f}\right)^{0.14} + 0.046\frac{\mathrm{Re}}{\varepsilon}\mathrm{Pr}(1-\varepsilon)^{2/3}. \qquad (12)$$

Relation (12) was verified over an extremely wide range of experimental conditions (including pressures): $1.4\cdot 10^2 \le \mathrm{Ar} \le 1.1\cdot 10^7$; $0.1 \le P \le 10.0$ MPa. This distinguishes it favorably from the majority of correlations known from the literature, for instance, [1, 2, 4].

2. Circulating fluidized bed. Due to the special features of the motion of the phases near the heat-transfer surface (the descending particle flux and the gas filtration at a relatively low rate that is weakly related to the mean filtration rate) the actual wandering velocity u_t for the given conditions is taken as u_w. Due to the relative smallness of the particle-concentration values the convection effect of the particles in the near-surface zone, i.e., the cofactor $(c_s/c_f)^{k_5}(\rho_s/\rho_f)^{k_6}$ in the first term of (11), can be neglected. With account for the unambiguous relation of $\mathrm{Re_t}$ and Ar Eq. (11) reduces to the form

$$\mathrm{Nu_{c-c}} = \frac{k_4 k_9}{k_1}\mathrm{Ar}^{k_2}(1-\varepsilon)^{k_3+k_{10}} + \frac{k_7^* k_9}{k_1}\mathrm{Ar}^{k_{11}}\mathrm{Pr}(1-\varepsilon)^{k_3-k_3^*+k_{10}}\left(\frac{d}{D}\right)^{k_8}. \qquad (11b)$$

In [8] we derived the following simple equation to describe conductive-convective heat transfer in a circulating fluidized bed:

$$\mathrm{Nu_{c-c}} = 1.65\,\mathrm{Ar}^{0.19}(1-\varepsilon)^{0.5} + 0.00049\,\mathrm{Ar}^{0.69}\mathrm{Pr}. \qquad (13)$$

Equation (13) was verified within rather wide ranges of experimental conditions: $0.1 \le P \le 5.0$ MPa; $0.058 \le d \le 0.827$ mm; $1 - \varepsilon \le 0.2$.

3. Vertical pneumotransport. Here the convection of the particles in the near-surface zone can also be neglected. The convective component of $\alpha_{c\text{-}c}$ is calculated using the well-known Mikheev formula [9]:

$$\mathrm{Nu_{conv}} = 0.021\,\mathrm{Re}^{0.8}\left(\frac{d}{D}\right)^{0.2}\mathrm{Pr}^{0.43}. \qquad (14)$$

In this case, model equation (11) acquires the form

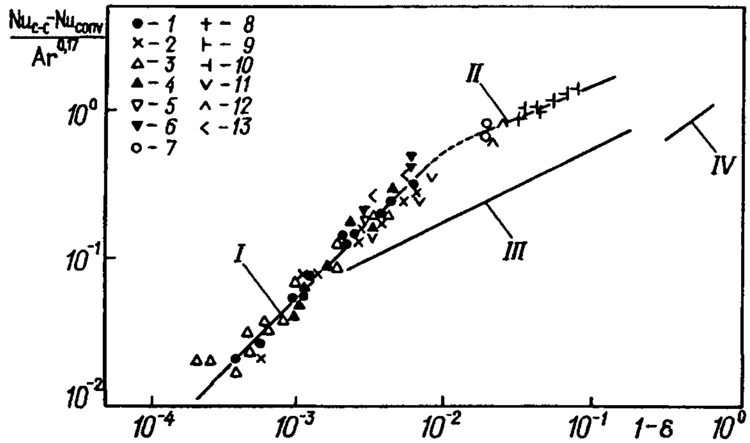

Fig. 1. Conductive heat transfer in disperse systems with suspended particles: 1-4) [10] ($u = 11.8$, 17, 20.8, 23.7 m/sec; $d = 0.076$ mm), 5-8) [5, Figs. 6-8] ($J_s/\rho_f u = 5$, 10 ($d = 0.14$), 30, 55 ($d = 0.165$, 0.15)); 9-10) [5, Figs. 6-11b] ($d = 0.165$, 0.14); 11-13) [5, Figs. 6-11a] ($u = 7.5$, 14.5 ($d = 0.15$), 15 ($d = 0.165$)); I) Eq. (15); II) (16); III) (13); IV) (12) ($\rho_s = 2500$ kg/m^3, $c_s = 800$ J/(kg·K)).

$$\mathrm{Nu}_{c-c} - \mathrm{Nu}_{conv} = \frac{k_4 k_9}{k_1} \mathrm{Ar}^{k_2} (1 - \varepsilon)^{k_3 + k_{10}} . \tag{11c}$$

Generalization of experimental data [5, 10] in accordance with (11c) yields the following relation

$$\mathrm{Nu}_{c-c} - \mathrm{Nu}_{conv} = 54.0\, \mathrm{Ar}^{0.17} (1 - \varepsilon) , \tag{15}$$

which is shown in Fig. 1 together with the experimental points. The domain of verification of Eq. (15) is: $1 - \varepsilon \le 10^{-2}$; $0.076 \le d \le 0.15$ mm; $12 \le u \le 42$ m/sec. At $1 - \varepsilon > 10^{-2}$ (the region of fluid-mixture flows [5]) the dependence on the particle concentration is smoother (Fig. 1) and the generalizng equation is

$$\mathrm{Nu}_{c-c} - \mathrm{Nu}_{conv} = 3.8\, \mathrm{Ar}^{0.17} (1 - \varepsilon)^{0.4} . \tag{16}$$

The particle concentration is determined from the solution of the transcendental equation

$$1 - \varepsilon = \bar{J}_s^* / (1 + \bar{J}_s^*) , \tag{17}$$

which follows from the definition of the specific flux of particles $J_s = \rho_s (1 - \varepsilon) v$ and the expression for their velocity [4] $v = (u - u_t^*(\varepsilon))/\varepsilon$. The quantity $u_t^*(\varepsilon)$ is calculated using the well-known Todes formula [4].

Figure 1 also shows functions (12) and (13). For correctness the conductive components of $\alpha_{c\text{-}c}$ are compared for the case of fine particles (for definiteness, with $\mathrm{Ar} \approx 200$). An analysis of the functions given in Fig. 1 allows us to draw the following conclusions:

1) the values of α_{cond} for fluid-mixture flows and a fluidized bed are approximately the same despite a considerable difference in the particle concentration;

2) the values of α_{cond} for a circulating bed asymptotically approach the values of α_{cond} for a fluidized bed with increase in $1 - \varepsilon$;

3) at the same values of $1 - \varepsilon$, α_{cond} for a circulating fluidized bed is approximately 1.5–2.5 fold lower than for pneumotransport conditions.

Figure 2 shows thicknesses of the boundary gaseous film calculated for different systems at $\mathrm{Ar} = 570$. For a circulating fluidized bed and pneumotransport $\lambda_{cond} = \lambda_f$ since the cofactors $(c_s/c_f)^{k_5} (\rho_s/\rho_f)^{k_6}$ in (4) can be neglected. In this case the formula for δ_f stemming from (8) is as follows:

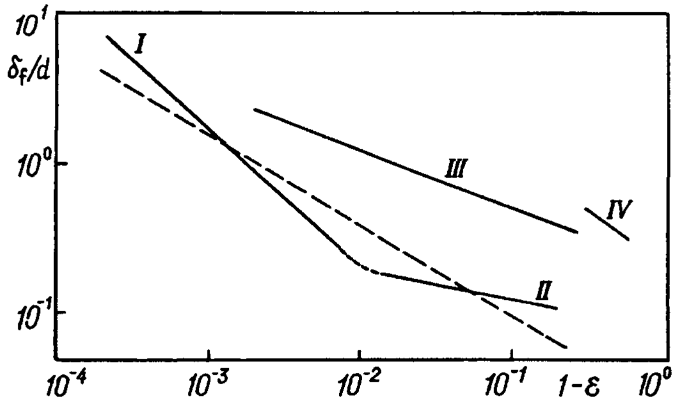

Fig. 2. Thickness of the boundary gaseous film, Ar = 570: I, II) pneumotransport (see Fig. 1); III) a circulating fluidized bed; IV) a fluidized bed; the dashed line is Eq. (20).

$$\delta_f/d = \Theta_s/\mathrm{Nu}_{cond}, \tag{18}$$

where Θ_s and Nu_{cond} are determined by (9), (13), (15), and (16). To calculate δ_f for a fluidized bed, the following formula [11] was used:

$$\delta_f/d = 0.38\ \mathrm{Ar}^{-0.1}(1-\varepsilon)^{-2/3}, \tag{19}$$

The same figure depicts results of an experimental study of δ_f in the form of the empirical relation

$$\delta_f/d = 0.0287\,(1-\varepsilon)^{-0.581}, \tag{20}$$

obtained in [12] in experiments with a circulating fluidized bed of sand with $d = 0.182$ mm (Ar = 570). As is seen, the experimental and calculated values of δ_f are in fair agreement under pneumotransport conditions. In the case of a circulating fluidized bed the agreement is only qualitative.

It is of interest to analyze the reasons for the considerable difference in the coefficients α_{cond} for the related systems of a circulating fluidized bed and pneumotransport. In terms of the film model these systems are characterized by different values of δ_f (Fig. 2). This can be attributable to the considerable difference in the velocities of the phases at the heat-transfer surface. Thus, according to data of [13], in a circulating fluidized bed of a catalyst ($d = 0.05$ mm) in all regimes with $u \le 6$ m/sec the velocity of the descending particle flux at the wall is 1–1.5 m/sec, while the ascending gas velocity is only ~0.5 m/sec. Obviously, for conditions typical of vertical pneumotransport, the level of the velocities of the phases directed upward is considerably higher, which results in lower values of δ_f (and, thus, in higher α_{cond}).

In conclusion, the suggested film model of heat transfer in disperse systems with suspended particles makes it possible to establish a general dependence of $\alpha_{c\text{-}c}$ on the basic governing factors, namely, formula (11). Correlations (12), (13), (15), and (16) of the same type, obtained based on it, describe a large amount of experimental data over wide ranges of experimental conditions and prove to be convenient for practical use.

NOTATION

$\mathrm{Ar} = gd^3(\rho_s/\rho_f - 1)/\nu_f^2$, Archimedes number; c, specific heat; D, apparatus (standpipe) diameter; d, particle diameter; g, gravitational acceleration; $\overline{J_s^*} = J_s/\rho_s(u - u_t^*)$; J_s, specific flux of particles; k_i, dimensionless coefficients; $\mathrm{Nu} = \alpha d/\lambda_f$, Nusselt number; P, pressure; $\mathrm{Re} = ud/\nu_f$, $\mathrm{Re}_w = u_w d/\nu_f$, Reynolds numbers; T, temperature; u_w, longitudinal gas velocity at the heat-exchange surface; u, gas filtration rate based on an empty section of the apparatus (standpipe); $u_t^*(\varepsilon)$, particle wandering rate under conditions of constraint; α, heat-transfer coefficient; δ_f, effective thickness of the gaseous film near at heat-transfer surface; ε, void content; $\Theta_s = (T_s - T_w)/(T_\infty - T_w)$; λ_f, molecular thermal conductivity of the gas; λ_{ef}, effective thermal conductivity of the gaseous film; ν_f, kinetic viscosity of the gas; ρ, density. Subscripts: cond, conductive; conv, convective, c-c, conductive-convective; f, gas, s, particles; w, heat-transfer surface; ∞, bed core; t, wandering conditions of a particle.

REFERENCES

1. D. Kunii and O. Levenspiel, Industrial Fluidization [Russian translation], Moscow (1976).
2. V. G. Ainshtein and A. P. Baskakov (eds.), Fluidization [in Russian], Moscow (1991).
3. I. M. Razumov, Fluidization and Pneumotransport of Loose Materials [in Russian], Moscow (1972).
4. O. M. Todes and O. B. Tsitovich, Apparatuses with a Fluidized Granular Bed [in Russian], Leningrad (1981).
5. Z. P. Gorbis, Heat Transfer and Fluid Mechanics of Disperse Straight-Through Flows [in Russian], Moscow (1970).
6. V. A. Borodulya, Yu. S. Teplitskii, I. I. Markevich, et al., Inzh.-Fiz. Zh., 58, No. 4, 597-604 (1990).
7. M. R. Golriz, in: Int. Symp. Eng. Foundation "Fluidization VIII," Tours, France (1995), pp. 185-192.
8. Yu. S. Teplitskii, Inzh.-Fiz. Zh., 71, No. 5, 799-804 (1998).
9. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Moscow (1981).
10. L. Farbar and M. J. Morley, Ind. Eng. Chem., 49, No. 7, 1143-1150 (1957).
11. V. A. Borodulya, Yu. S. Teplitskii, I. I. Markevich, et al., "Heat and mass transfer between a fluidized bed and a surface under normal and extreme conditions" (Preprint No. 2 of the Heat and Mass Transfer Institute of the BSSR Academy of Sciences), Minsk (1989).
12. M. C. Lints and L. R. Glicksman, in: Preprints of CFB-4, Somerset (1993), pp. 350-355.
13. W. Nowak, "A conprehensive study of the circulating fluidized bed" (A thesis presented to qualify as assistant professor), Chęstochowa (1989).